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Field Theory, Curdling, Limit Cycles, and Cellular Automata

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It is suggested that the process of curdling is an important question for the science of fractals. A field equation which displays nucleation (curdling) of particles out of a pure radiation field is discussed. The particle formation arises naturally from the nonlinear character of the equation rather than from imposed quantization conditions. The relativistically invariant equation is

$$\operatorname{div}(\rho^{\mu}(\mathbf{r}, t, \mathbf{\Omega}_1)) = \int [\rho_{\mu}(\mathbf{r}, t, \mathbf{\Omega}), \rho^{\mu}(\mathbf{r}, t, \mathbf{\Omega}_2)]\, d\mathbf{\Omega}_2$$

where $[\ ,\ ]$ denotes commutator. $\rho^{\mu}(\mathbf{r}, t, \mathbf{\Omega})$ is both a 4-vector and a 2×2 matrix. It represents substance at r, t traveling with the velocity of light in direction $\mathbf{\Omega}$. A unique feature is that the scattering of $\rho(\Omega_1)$ by $\rho(\Omega_2)$ as determined by the right-hand side of the above equation results in fields that persist at a given place even though ρ itself represents substance traveling always at the speed of light. Explicit solutions are given for the case of one dimension. Fields representing particles are obtained and shown to have specially oscillatory structure with incipient fractal character.

KEY WORDS: Curdling of fields; field theory; fiber bundle physics.

1. INTRODUCTION

If one scans the titles and abstracts of the contributions for this fractals conference one sees a large, almost exclusive, preoccupation with fractal dimensions. But for this author the most significant concept presented in *The Fractal Geometry of Nature* is the concept of curdling. As Mandelbrot observes there is curdling at various levels in our hierarchical universe.[1] Not only do we get curds and whey separating from the more uniform milk which precedes it, but we get stars condensing out of preexisting clouds of

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matter whose dimensions are on the order hundreds of light years across. At a higher level we have galaxy formation from the primal cloud of hydrogen and helium believed to have resulted from the Big Bang. Notice that a given level in the hierarchy is a nest for the large number of elements of the hierarchy immediately below it. Thus the universe is a nest for the hundred billion galaxies that comprise it. Each galaxy is a nest for the billions of stars and solar systems that comprise it. Each solar system is a nest for the millions of species of animals and plants that inhabit it. Each species usually contains millions to billions of members. Each animal or plant is composed of (usually) billions of cells. Each cell contains millions and sometimes billions of macromolecules. Each macromolecule contains millions or at least thousands of atoms. And finally each atom is a nest of fundamental particles.

To speak of fractals is to speak of the interrelationships of the elements within a given level (nest) of the hierarchy. It seems sensible that the process of formation of these elements (the process of curdling) has a bearing on these interrelationships. Thus the study of curdling has a logical priority over the study of fractals.

For the physicist perhaps the most fundamental question that can be asked is how do the fundamental particles from which all the nests of the hierarchies of matter are constructed condense or nucleate from their primal field? It is the viewpoint of this physicist that the question of the curdling of the field into those nodules of matter that we call particles is also the preeminent question for the science of fractals, and it is this question that we shall address.

There is a view of the world, of which Einstein was the main protagonist, which holds that particles arise from continuum field equations and are due to the nonlinear character of the equations. The particles can be associated either with singularities of the field or with regions of high field intensity. A prime characteristic of the particle is that it persists in time. In this view quantization arises from the nonlinearity of the field equations. It is a historical fact that this view has so far been unsuccessful.

There is another view of the world, of which Bohr was the main protagonist, which simply accepts the existence of particles and then goes on to formulate laws of interaction among the particles. These laws of interaction are so chosen that the nest formed by the particles is properly described. However, quantization does not arise naturally from the equations which are linear but rather is imposed from the outset. It is a historical fact that this view of the world has had continuing success. A successful example of the latter view is Schrödinger's equation.

In this paper we espouse the former view. Four points need to be made. (1) The nucleation or curdling of the primal field to produce particles is an essential component of the nonlinear field theorist's (NLFT) dream. The hope is that the nonlinearities of the field equation will result in particles without preimposed quantum conditions. (2) Quantum theory is so successful that NLFT must be able to derive or reproduce it. (3) NLFT particles must be stable to perturbations of the field. Their structure must correspond in some rough way to solitons and their motion to the behavior of particles near attractors or limit cycles. [Our equations in $(1+1)$ dimensions are similar to the ordinary soliton equations with the difference that the nonlinear term contains derivatives of the dependent variable as well as the dependent variable itself. Compare our second-order equations to those in the book by Rajaraman.[2]] (4) Relativity has removed action at a distance and allows only for contiguous action. The totality of possible physical laws are thereby greatly reduced, but there is still much structure possible in fields obeying the principle of contiguous action. This is consistent with the mathematical treatment of cellular automata which shows that very simple rules for the interaction of nearest-neighbor beads can result in complex spatial and temporal structures.[3] The above four points serve to explain the title of this paper. We shall touch on each of them throughout the paper.

There are three objectives of this paper: (1) to describe the basic idea behind a class of simple and easily understood nonlinear field theories; (2) to display the simplest version of the theory [it is a $(1+1)$-dimensional theory] and show how it results in particles nucleating from the primal field; (3) to suggest an equation for the real world of $(3+1)$ dimensions.[4] We will examine the solutions of this equation in $(1+1)$ dimensions and show how they relate to the four points of the previous paragraph.

2. THE BASIC IDEA

Our basic assumption is that all matter can be represented by a single primal radiation field $\rho(\mathbf{r}, t, \mathbf{\Omega})$ which we describe by reference to Fig. 1. $\rho(\mathbf{r}, t, \Omega_1)$ represents substance traveling with velocity c in direction Ω_1 at the space-time point (r, t). The only way a given pencil of this substance can change is if it is scattered by other substance $\rho(\mathbf{r}, t, \Omega_2)$ at the same space-time point that is traveling in another direction. Let us suppose that we have a given concentration of this field at a given place. If there were no scattering the field would dissipate explosively (leave the vicinity with velocity c). The question we ask is whether we can find scattering laws which allow certain field configurations to persist in a given place for long times. Such field configurations if they exist will be called particles. Symbolically such a law would look like

$$\frac{D(\mathbf{r}, t, \mathbf{\Omega})}{Dt} = \int f(\rho(\mathbf{\Omega}_1), \rho(\mathbf{\Omega}_2))\, d\mathbf{\Omega}_2 + \iint g(\rho(\mathbf{\Omega}_1, \rho(\mathbf{\Omega}), \rho(\mathbf{\Omega}_3))\, d\mathbf{\Omega}_2\, d\mathbf{\Omega}_3 \quad (1)$$

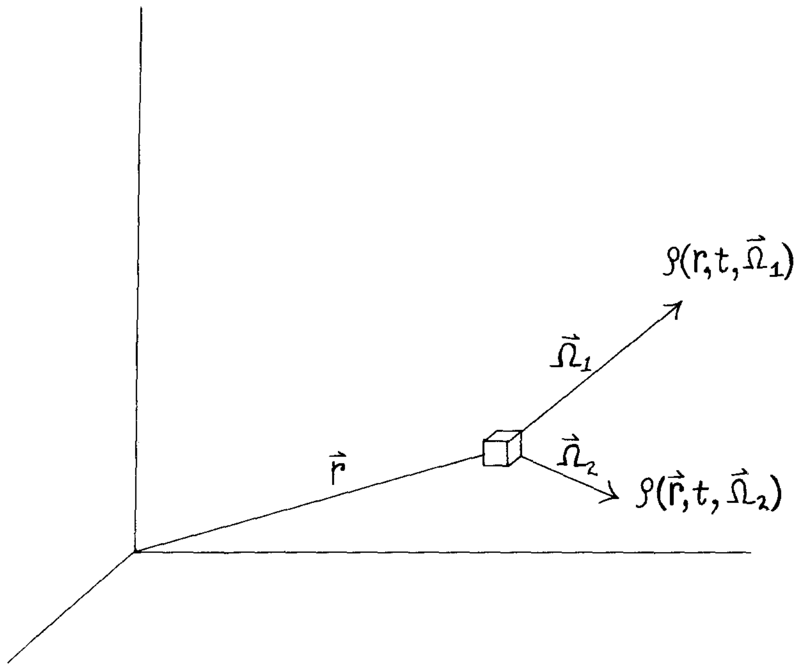

Fig. 1. Matter is represented by a radiation field $\rho(\mathbf{r}, t, \mathbf{\Omega})$ which gives the amount ρ of primal field at $\mathbf{r}$, t traveling in direction $\mathbf{\Omega}$ with the speed of light c. The only way that $\rho(\mathbf{\Omega}_1)$ can change is by being scattered by $\rho(\mathbf{\Omega}_2)$ (the integral of ρ over Ω_2) at each space-time point $\mathbf{r}, t$. The resulting equations for the rate of change of $\rho(\mathbf{r}, t, \mathbf{\Omega})$ look very much like Boltzmann's equations but lead to a curdling of the field rather than smoothing. Curds (regions of high field intensity) exist which persist in time and are stationary in space. They represent particles.

where the left-hand side represents the comoving derivative, the first term of the right-hand side represents binary scattering of $\rho(r, t, \Omega_1)$ by all of the substance traveling through that point, while the second term represents substance originally in directions 2 and 3 that scatters into direction 1. g may, but need not be dependent on $\rho(\Omega_1)$. The question is, can we find scattering laws f, g so that certain field configurations maintain themselves in existence at a given place? These fields will then represent the fundamental particles. Notice that according to this scheme a particle is always an extended object, but every part of the field that comprises the particle is always moving with the velocity of light c, even though the particle may be stationary in space.

Two questions immediately suggest themselves. (1) Can one find examples of f and g that will in fact result in stationary particles? (2) Why assume that the ρ field always moves with the velocity of light?

The answer to the first question is yes and we will devote the rest of the paper, starting with Section 3 to displaying explicit examples. First we shall in Section 3 solve the simplest field equations possible for a world of $(1+1)$ dimensions and describe the resulting particle formation. Second we shall in Section 4 treat the $(1+1)$-dimensional version of the equation obtained previously[4] for the real world of $(3+1)$ dimensions. Particles also will be displayed for this case.

The answer to the second question is that the assumption that the primal field is a radiation field will stand or fall on its own merits; it nonetheless is the result of a cogent argument given previously.[5] Concerning this assumption we merely note the following. (1) It is a manifestly covariant idea since anything that moves with velocity c in one coordinate system moves with velocity c in all coordinate systems (we shall confine ourselves to the Poincaré group for simplicity). (2) It results in equations much simpler than if we assumed the only other covariant possibility, which is that ρ can take on all velocities from 0 to c. (3) When a particle exists or is created its existence becomes manifest simply from the fact that ρ persists at a given place. (4) Because the scattering law (the right-hand side of the equation) is at least quadratic in ρ the particles will be quantized by virtue of the non-linearity of the field equations.

3. THE SIMPLEST EQUATION POSSIBLE IN (1 + 1) DIMENSIONS

In one spatial dimension we can have the primal field traveling to the right with velocity c and/or we can have the primal field traveling to the left with velocity c. Since the only way ρ can change is by scattering we must have

$$\begin{aligned} \frac{\partial \rho_R}{\partial t} + c\frac{\partial \rho_R}{\partial x} &= f(\rho_R, \rho_L) = \rho_R\,\rho_L \\ \frac{\partial \rho_L}{\partial t} - c\frac{\partial \rho_L}{\partial x} &= -f(\rho_R, \rho_L) = -\rho_R\,\rho_L \end{aligned} \tag{2}$$

We have given the simplest possible form to f which is relativistically covariant. The minus sign is chosen so that what is scattered out of ρ_R is scattered into ρ_L and conversely. Because the comoving derivative is also a divergence the equations are invariant to the inhomogeneous Lorentz transformation. The general solution to these equations is known.[5] We shall simply report on two solutions corresponding to particles. The single-particle case is given by $(c = 1)$

$$\rho_R = \rho_L = -1/x \tag{3}$$

The field represents a particle stationary in time even though it is composed entirely of stuff traveling with the velocity of light c. It is quantized. Note that A/x is a solution only for $A=-1$. By performing a boost we obtain a particle moving with velocity v $(-1<v/c<+1)$.

The solution representing two particles is

$$\rho_R=-2(x-t)/(x^2+t^2-A^2), \qquad \rho_L=-2(x+t)/(x^2+t^2-A^2) \tag{4}$$

as can be verified by inspection. This solution represents two particles created at time $t=-A$ at location $x=0$. They separate and at $t=0$ they have their maximum separation of $2A$. They then fall back onto themselves annihilating themselves at time $t=A$ (and $x=0$). The general solutions of Eq. (2) are known[5] and they allow for the existence of n particle states. In this simple one-dimensional world, particles are created and annihilated in pairs.

The stability of the solutions can be discussed via a development that follows the spirit of Ljapunov.[6] If we write

$$\rho_R=-1/x+\delta_R, \qquad \rho_L=-1/x+\delta_L \tag{5}$$

then substituting into Eq. (2) and retaining terms to first order in δ_R, δ_L we obtain.

$$\begin{aligned}\frac{\partial\delta_R}{\partial t}+\frac{\partial\delta_R}{\partial x}&=-(\delta_R+\delta_L)/x\\ \frac{\partial\delta_L}{\partial t}-\frac{\partial\delta_L}{\partial x}&=+(\delta_R+\delta_L)/x\end{aligned} \tag{6}$$

which are linear equations for δ_R, δ_L. The substitution

$$\frac{\partial F}{\partial x}=\delta_R+\delta_L, \qquad \frac{\partial F}{\partial t}=\delta_L-\delta_R \tag{7}$$

leads to

$$\frac{\partial^2 F}{\partial t^2}-\frac{\partial^2 F}{\partial x^2}=2\,\frac{\partial F}{\partial x}\Big/x \tag{8}$$

These equations being linear are amenable to classical techniques.

4. SOME SYMMETRIES

If $\rho(x,t)$ is a solution then $k\rho(kx,kt)$ is also a solution. We call this the scaling group.

The following four elements form the Abelian 4-group:

I	$\rho_R = +\rho_R(x, t)$,	$\rho_L = +\rho_L(x, t)$
O_1	$\rho_R = -\rho_R(-x, -t)$,	$\rho_L = -\rho_L(-x, -t)$
O_2	$\rho_R = +\rho_L(x, -t)$,	$\rho_L = +\rho_R(x, -t)$
O_3	$\rho_R = -\rho_L(-x, t)$,	$\rho_L = -\rho_L(-x, t)$

These groups along with the Poincaré group can be used to generate many solutions from one solution. Notice that the isolated particle $\rho_R = \rho_L = -1/x$ has the highest symmetry possible. Each element of the scaling group and of the 4-group generates $-1/x$ from $-1/x$. The existence of the scaling group suggests that particles are represented by solutions that are homogeneous of degree -1. Otherwise we would not have quantization of the field.

4.1. A Candidate Equation for the Real world of (3 + 1) Dimensions

An equation has been suggested for the real world.[4] It is

$$\frac{\partial \rho^{\mu}(\Omega_1)}{\partial x^{\mu}} = \int [\rho_{\mu}(\Omega_1), \rho^{\mu}(\Omega_2)]\, d\Omega_2 \tag{9}$$

where the left-hand side is the four divergence and [,] represents the commutator. ρ again represents substance moving with velocity c but it is now a 2×2 matrix rather than just a number. There are a number of interesting properties of this equation.

(1) Manifest relativistic invariance of the equations.

(2) Principle of contiguous action is satisfied.

(3) There is a conservation law on the amount of ρ; see Eq. (11).

(4) The field quantity ρ always moves with velocity c.

(5) The form of Eq. (9) is invariant to six separate symmetry groups.[4]

(6) Existence of particles is nothing more than persistence of ρ at a place in space.

(7) They are the simplest nontrivial equations possible for $(3 + 1)$ dimensions.

Any choice of $\rho^{\mu}(x^{\nu}, \Omega)$ is a possible initial condition and Eq. (9) determines how ρ^{μ} evolves (curdles) in time. However, it is not true that any choice of ρ can be the result of an evolution in time since the equations are not invariant to time reversal. Examples of fields that are *a priori* possible but which can never come to be are easily contrived.[2] We shall use Eq. (9) unless the demands of the real world force us to complicate it. The connection to known physics is made via the relation

$$A^{\mu}(r, t) = \int \rho^{\mu}(r, t, \boldsymbol{\Omega})\, d\boldsymbol{\Omega} \tag{10}$$

where A^{μ} is the electromagnetic 4-potential.

Integration of Eq. (9) over $\boldsymbol{\Omega}$ immediately gives the Lorentz condition,

$$\frac{\partial A^{\mu}}{\partial x^{\mu}} = 0 \tag{11}$$

which is a conservation law on A. Three other conservation laws on integrals of ρ over $\boldsymbol{\Omega}$ have been identified with the conservation of energy, "spin," and charge.[4]

4.2. Application to (1 + 1) Dimensions

Since ρ is a 2×2 matrix we can without loss of generality expand it with the Pauli matrices as a basis:

$$\rho^{\mu} = A^{\mu} I + B^{\mu} \sigma_x + C^{\mu} \sigma_y + D^{\mu} \sigma_z \tag{12}$$

In terms of A and $\mathbf{F} = (B, C, D)$, Eq. (9) becomes for $(1 + 1)$ dimensions

$$\frac{\partial A_R}{\partial t} + \frac{\partial A_R}{\partial x} = 0, \qquad \frac{\partial A_L}{\partial t} - \frac{\partial A_L}{\partial x} = 0 \tag{13}$$

$$\frac{\partial \mathbf{F}_R}{\partial t} + \frac{\partial \mathbf{F}_R}{\partial x} = 2\mathbf{F}_R \times \mathbf{F}_L, \qquad \frac{\partial \mathbf{F}_L}{\partial t} - \frac{\partial \mathbf{F}_L}{\partial x} = 2\mathbf{F}_L \times \mathbf{F}_R \tag{14}$$

We shall ignore the A part of the field since it is completely uncoupled from the F part and since it, suffering no scattering, cannot represent stationary particles. The cross-product on the right-hand side shows that $\mathbf{F}_R$ precesses around $\mathbf{F}_L$ while simultaneously $\mathbf{F}_L$ precesses around $\mathbf{F}_R$. This allows for spatial and temporal oscillations in the field and allows us to associate a wavelength to each particle. The substitution $\mathbf{F}_R + \mathbf{F}_L = \partial G/\partial x$, $\mathbf{F}_L - \mathbf{F}_R = \partial G/\partial t$ results in

$$\frac{\partial^2 \mathbf{G}}{\partial x^2} - \frac{\partial^2 \mathbf{G}}{\partial t^2} = 2\frac{\partial \mathbf{G}}{\partial x} \times \frac{\partial \mathbf{G}}{\partial t} \tag{15}$$

[2] Two examples are (1) a live rose encased in diamond and (2) a man in a space ship farther than his lifetime from any possible progenitor.

and using $v = x - t$, $u = x + t$ results in

$$\frac{\partial^2 \mathbf{G}}{\partial u\, \partial v} = \frac{\partial \mathbf{G}}{\partial v} \times \frac{\partial \mathbf{G}}{\partial u} \tag{16}$$

which are the canonical forms of interesting equations that seem not to have been previously investigated. Of course Eqs. (16), (15) and (14) have the same information content.

Although we have not been able to solve these equations completely certain solutions corresponding to particles have been obtained and we shall spend the rest of this paper discussing them. A time-independent solution of Eq. (14) can be obtained by use of $\mathbf{F}_R = \mathbf{F}_L + \mathbf{C}$. We obtain

$$\frac{d\mathbf{F}_L}{dx} = 2\mathbf{C} \times \mathbf{F}_L \tag{17}$$

which is identical in form to the equation for precession of a magnetic dipole in a magnetic field and has as its solution[7]

$$\begin{aligned} F_L &= M[\hat{\imath}\cos(2C_3 x + \varphi) + \hat{\jmath}\sin(2C_3 x + \varphi)] + \hat{k}N_3 \\ F_R &= M[\hat{\imath}\cos(2C_3 x + \varphi) + \hat{\jmath}\sin(2C_3 x + \varphi)] + \hat{k}(N_3 + C_3) \end{aligned} \tag{18}$$

where $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ are mutually perpendicular unit basis vectors in "spin" space and we have placed $\mathbf{C}$ in the $\hat{k}$ direction. Although our solutions are real we note that F_L and F_R need not be limited to the real number field. Equation (18) represents a particle in which F_L and F_R precess about the spin direction $\hat{k}$ with a spatial frequency $2C_3$. We now have a plethora of particles since M, N, and C_3 can be varied continuously. Thus, the equation serves to amply justify our assertion that stationary particles can be composed entirely of a primal radiation field which is always moving with velocity c. However, since M, N, and C_3 can be varied continuously and still solve Eqs. (14), we do not have quantized particles as we did have in Section 3. If this feature were to persist in $(3 + 1)$ dimensions we would need to add other terms to the right-hand sides of Eqs. (14) in an attempt to recover quantization.

A most interesting aspect of Eq. (18) is that we can associate a frequency ω with each particle that is proportional to the particles velocity. This is a direct result of the spatial periodicity. If we were to move with velocity v relative to the particle then the number of periods traversed in a unit time would be proportional to v (and to C_3). To obtain the exact formula for ω we first note that $\mathbf{F}_R$ and $\mathbf{F}_L$ in an unprimed coordinate system are related to $\mathbf{F}_R$ and $\mathbf{F}_L$ in a primed coordinate system moving with relative velocity v by[5]

$$\mathbf{F}_L = \frac{(1+v/c)^{1/2}}{(1-v/c)^{1/2}} \mathbf{F}'_L, \qquad \mathbf{F}_R = \frac{(1-v/c)^{1/2}}{(1+v/c)^{1/2}} F'_R$$
$$x = \gamma(x' - vt') \tag{19}$$

Thus the primal field of a particle moving with velocity v is

$$\begin{aligned} \mathbf{F}'_L &= (1-v/c)\,\gamma[M\hat{\imath}\cos(2c_3\gamma[x'-vt'] + \varphi) \\ &\quad + M\hat{\jmath}\sin(2c_3\gamma[x'-vt'] + \varphi) + \hat{k}N_3] \\ F'_R &= (1+v/c)\,\gamma[M\hat{\imath}\cos(2c_3\gamma[x'-vt'] + \varphi) \\ &\quad + M\hat{\jmath}\sin(2c_3\gamma[x'-vt'] + \varphi) + \hat{k}(N_3 + C_3)] \end{aligned} \tag{20}$$

where $\gamma = 1/[1-(v/c)^2]^{1/2}$. Thus ω' is given by

$$\omega' = 2c_3 v/[1-(v/c)^2]^{1/2} \tag{21}$$

Notice that ω' is to high accuracy proportional to the velocity of the particle.

In quantum mechanics one assigns to a particle a wavelength which is inversely proportional to its velocity. This receives a simple interpretation in our scheme. A particle has small wavelength at high velocity simply because the number of maxima in the field F (there is one per period) passing a given point is proportional to the velocity. Having made the assertion that particles are fields extended in space we are forced to the conclusion that they have a spatial and temporal periodicity. This conclusion is strengthened by the following consideration. Imagine the particle to be stationary (for concreteness imagine an electron) and pass a crystal across it with velocity v. Even though the electron was not moving it scatters from the crystal as if it (the electron) had a wavelength associated with it. This is consistent with the view that the particle is an extended object and that it has a spatially periodic structure even when it is stationary.

The oscillatory structure of the fields is a consequence of the fact that ρ is a 2×2 matrix.

Our picture of the wave properties of the particles is not complete because we have not yet determined the structure of any real particle. We have in fact only the most rudimentary hints. The author feels somewhat like a hounddog who has the scent but has not yet found the quarry.

Another interesting solution is that which is homogeneous of degree -1.

Use of

$$\mathbf{F}_R = \frac{\mathbf{f}_R(\theta)}{x}, \qquad \mathbf{F}_L = \frac{\mathbf{f}_L(\theta)}{x}, \qquad \theta = t/x \tag{22}$$

leads to the pair of coupled ordinary differential equations

$$\frac{d[(1-\theta)\,\mathbf{f}_R]}{d\theta} = \mathbf{f}_R \times \mathbf{f}_L, \qquad \frac{d[(1+\theta)\,\mathbf{f}_L]}{d\theta} = \mathbf{f}_L \times \mathbf{f}_R \tag{23}$$

which are easily solved to obtain

$$\begin{aligned}
\mathbf{F}_R &= \frac{M}{x-t}\left[\hat{\imath}\cos\left(c_3 \ln\left[\frac{x+t}{x-t}\right] + \varphi\right) + \hat{\jmath}\sin\left(c_3 \ln\left[\frac{x+t}{x-t}\right] + \varphi\right)\right] \\
&\quad + \frac{\hat{k}N_3}{x-t} \\
\mathbf{F}_L &= -\frac{M}{x+t}\left[\hat{\imath}\cos\left(c_3 \ln\left[\frac{x+t}{x-t}\right] + \varphi\right) + \hat{\jmath}\sin\left(c_3 \ln\left[\frac{x+t}{x-t}\right] + \varphi\right)\right] \\
&\quad + \frac{\hat{k}(C_3 + N_3)}{x+t}
\end{aligned} \tag{24}$$

These equations also have a spatial frequency but the frequency is a fuction of position and becomes infinite at $x = \pm t$. These curves are not fractal curves since only these two points have (a semblence of) fractal character. However, it is not out of the realm of possibility that in two or three dimensions these accumulation points of fractal character will translate to accumulation lines or surfaces of fractal character.

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